

## A NEUTRON-DIFFRACTION STUDY OF THE FERRIC TOURMALINE, BUERGERITE<sup>1</sup>

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### ABSTRACT

The structural parameters of buergerite (ideally  $\text{NaFe}_3\text{B}_3\text{Al}_6\text{Si}_6\text{O}_{30}\text{F}$ ) have been refined using three-dimensional, single-crystal neutron diffraction data. The occupancy factors in the cation sites have also been refined, and, in agreement with the X-ray results, a substitution of Fe into the  $18c$  Al point position of space group  $R3m$  is found.

A difference synthesis resulted in the location of a hydrogen atom in the structure—corresponding to the replacement of O by OH in one site of the  $\text{FeO}_6$  octahedron about 8 percent of the time.

Because of the differences in scattering amplitudes, some of the bond distances have been determined to slightly higher precision than in the X-ray determination. The final  $R$  ( $R^2$ ) factor for 391 independent reflections is 0.034. The maximum error in the final difference synthesis is  $0.4 \text{ fm } \text{\AA}^{-3}$ —about 1 percent of the maximum peak height in the observed scattering density map. The position and thermal parameters agree extraordinarily well with those of the X-ray study, and confirm the results of that work in detail; the precision of the atomic position determinations is about  $0.003 \text{ \AA}$  in both studies.

A comparison of neutron powder diffraction patterns at  $300^\circ\text{K}$  and at  $4.2^\circ\text{K}$  provides no evidence for magnetic ordering or other structural change between these temperatures.

### INTRODUCTION

Buergerite, a ferric tourmaline with the idealized formula  $\text{NaFe}_3\text{B}_3\text{Si}_6\text{Al}_6\text{O}_{30}\text{F}$ , has been described by Mason, Donnay, and Hardie (1964) and by Donnay, Ingamells, and Mason (1966). In a recent X-ray diffraction study Barton (1969) found the crystal structure of buergerite to be very similar to the dravite structure of Buerger, Burnham, and Peacor (1962). An important result of the X-ray study is that some iron replaces aluminum in the  $18c$  position of space group  $R3m$ , a substitution not demanded by the chemical analysis. The ratios of atomic scattering amplitudes for neutrons and X-rays differ; therefore a neutron diffraction study should provide independent evidence of cation substitution. Furthermore, neutron diffraction data are sensitive to the ordering of unpaired electrons, and a report of the antiferromagnetic behavior of buergerite (Tsang, Thorpe, Senftle, and Donnay, 1970) gave further stimulus to the study reported in this paper.

<sup>1</sup> Research performed under the auspices of the U. S. Atomic Energy Commission.

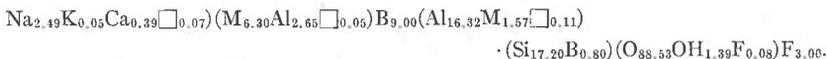
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## CRYSTAL DATA

The cell constants, density, and space group were taken as those found in the X-ray study:  $a = 15.869 \text{ \AA}$ ,  $c = 7.188 \text{ \AA}$ ,  $\rho_x = 3.29 \text{ g cm}^{-3}$ ,  $R3m$ .

## EXPERIMENTAL

The crystalline material used in this investigation was kindly supplied by Dr. John S. White of the U. S. National Museum (specimen R-12631), and was from the same locality as the crystals studied in the investigations referenced above. The final formula adopted by Barton on the basis of the chemical analysis and the x-ray structure determination is



with  $\text{M} = \text{Fe}_{0.951} + \text{Ti}_{0.028} + \text{Mg}_{0.013} + \text{Mn}_{0.008}$ . This overall chemical composition was assumed in the present study, and the distribution of the cations was essentially confirmed.

A doubly terminated prismatic crystal was chosen for data collection. The distance between the two monohedra and between the three pairs of opposite prism faces (10 $\bar{1}$ 0, 01 $\bar{1}$ 0, 11 $\bar{2}$ 0) were each equal to 1.90 mm. The crystal volume was 6.25 mm<sup>3</sup>. The crystal was mounted with the [110] axis parallel to the  $\phi$  axis of a four-circle goniometer at the Brookhaven National Laboratory High Flux Beam Reactor. A monochromatic beam of neutrons with wavelength 1.047  $\text{\AA}$  was obtained by reflection from the (111) face of a single crystal of copper; at this wave-length, the amount of second order contamination in the beam is negligible.

Integrated intensities were obtained for reflections with  $(\sin\theta)/\lambda < 0.65$  by a  $\theta$ - $2\theta$  step scan-method, with steps taken every 0.1 degree in  $2\theta$ . The length of the scan was a variable function of scattering angle, chosen to obtain several background points at each end of the scan. The data were reduced to squared structure amplitudes by background correction, application of the Lorentz factor  $\sin 2\theta$ , and correction for absorption ( $\mu = 4.36 \text{ cm}^{-1}$ ). The transmission factors varied from 0.46 to 0.52. Most reflections were observed twice; 822 separate ( $hkl$ ) values were obtained from 1644 measurements. The agreement factors  $R$  and  $R_w$  for the 585 multiply-observed reflections were 0.023 and 0.025.<sup>1</sup> Only seventeen of these reflections deviated by more than  $4\sigma$  from the weighted mean.

The intensities for the 822 ( $hkl$ ) values were now averaged over sets of symmetry-equivalent reflections to obtain 397 independent reflections. The values of  $R$  and  $R_w$  for this averaging were 0.024 and 0.025; no departure from Laue symmetry  $\bar{3}m$  was suggested. For the subsequent analysis, each reflection was assigned an estimated standard deviation which was the highest of the following three values: (1) the counting statistical error, (2) the standard deviation of the mean as estimated from the agreement between equivalent reflections, (3) five percent of  $F^2$ .

## STRUCTURE REFINEMENT

Initial least squares refinement of all position parameters, anisotropic thermal parameters, an isotropic extinction parameter (Zachariasen, 1967), a scale factor, and atomic scattering lengths for all positions ex-

$$^1 R = \frac{\sum_{hkl,j} |F_j^2(hkl) - F_\mu^2(hkl)|}{\sum_{hkl,j} F_\mu^2(hkl)}$$

$$R_w = \left[ \frac{\sum_{hkl,j} \omega_j |F_j^2(hkl) - F_\mu^2(hkl)|^2}{\sum_{hkl,j} F_\mu^4(hkl)} \right]^{1/2}$$

with the weights  $\omega_j = 1/\sigma_j^2$  being determined by Poisson counting statistics.

TABLE 1. NEUTRON SCATTERING LENGTHS IN  $10^{-12}$  CM  
FOR THE ELEMENTS IN BUERGERITE

Those values with standard deviations in parentheses were refined in this study,  
and mean values are based on the chemical composition given by Barton.

Atom	Atomic Number	b
Na	11	0.351
K	19	0.37
Ca	20	0.49
$\mu(\text{Na site})$	—	0.361
Fe	26	0.95
Ti	22	-0.34
Mg	12	0.52
Mn	25	-0.36
$\mu(M)$	—	0.898
Al	13	0.35
B	5	0.540 (9)
O	8	0.577
F	9	0.55
H	1	-0.378
Fe site	—	0.763 (11)
Al site	—	0.385 (7)

cept the 18c Al position (which was normalized to agree with Barton's results) indicated that the chemical composition given by Barton and the neutron scattering lengths given in Table 1 are indeed satisfactory. In subsequent refinements, most scattering lengths were fixed at their theoretical values. Exceptions were made for the 18c Al position and the 9b Fe position; the scattering lengths at these positions were varied to obtain an independent estimate of the (Al, Fe) replacement found by Barton.<sup>1</sup> The boron scattering length was also refined, inasmuch as it is not as well characterized by previous crystallographic studies. The O(1) site was assumed to be entirely F; the scattering lengths for F and O are nearly equal, and a moderate substitution of O for F would not affect the results.

Refinement of position, thermal, scale, and extinction parameters along with the three scattering lengths reduced the value of  $R$  and  $R_w$  (both based on  $F^2$ ) to 0.036 and 0.061. The standard deviation of an observation of unit weight ( $S$ ) was 1.276. Observed and difference scattering density syntheses were calculated at this stage. Peak heights in the observed map ranged from  $44.96 \text{ fm} \cdot \text{\AA}^{-3}$  for Fe to  $21.25 \text{ fm} \cdot \text{\AA}^{-3}$  for

<sup>1</sup> The restraint on the 18c Al scattering length in the previous refinement does not affect the results of the final refinement; effectively, in the final refinements, normalization was made to the scattering factor for oxygen, which is well known.

Na (one Fermi or femtometer (fm) =  $10^{-15}$  m). Figure 1 shows a section of this map in a plane near the plane of the  $\text{BO}_3$  triangle. The maximum peak heights in the difference map were  $0.42 \text{ fm} \cdot \text{\AA}^{-3}$ , and there were many peaks nearly this high scattered uniformly throughout the cell. Thus the noise level may be taken as about  $0.4 \text{ fm} \cdot \text{\AA}^{-3}$ . Negative peaks (see Figure 2), which will correspond to hydrogen atoms, were found above this noise level at only two chemically reasonable positions (about  $1 \text{ \AA}$  from oxygen sites) and not too close to other atoms. The largest peak occurred at  $(-0.13, +0.13, 0.39)$  with a density of  $-0.99 \text{ fm} \cdot \text{\AA}^{-3}$ ; this is approximately  $1 \text{ \AA}$  from O(3) along the O(3)–O(5) line. A second peak with a density of  $-0.59 \text{ fm} \cdot \text{\AA}^{-3}$  was found at  $(0.00, 0.00, -0.12)$ . This is about  $1 \text{ \AA}$  from the O(1)  $\equiv$  F site. A peak with a density of  $-0.61 \text{ fm} \cdot \text{\AA}^{-3}$  was found about  $1 \text{ \AA}$  from B and  $1.8 \text{ \AA}$  from  $\text{O}_8$ ; a peak with density  $-0.55 \text{ fm} \cdot \text{\AA}^{-3}$  was found at about  $1 \text{ \AA}$  from O(5) but at  $1.8 \text{ \AA}$  from Si; three other peaks with densities between  $0.40$  and  $0.48 \text{ fm} \cdot \text{\AA}^{-3}$  were either too close to Fe or a long distance from any atom; all these peaks were judged to be either noise or diffraction ripple from the heavy atoms. Least squares refinement of the positions and occupancy factors for the two reasonable hydrogen atom sites led to occupancies of  $0.083(11)$  for H(03) in a  $9b$  position and  $0.006(20)$  for H(F) in a  $3a$  position. Thus the refinement would seem to indicate essentially no hydrogen in the latter position. After the final cycles of least squares refinement including only the H(03) hydrogen atom, a final difference synthesis now had its largest peak at the position  $(0, 0, -0.14)$  with a density of  $-0.5 \text{ fm} \cdot \text{\AA}^{-3}$ , but there was a steady gradation of noise peaks falling off from this level. Although charge-balance considerations (Donnay, 1970) suggest significant hydrogen occupancies in both positions, the ionic disorder in both the  $\text{Na}^+$  and  $\text{F}^-$  positions on the three-fold axis (as evidenced by the high thermal parameters) may also smear out the hydrogen atom so that it is not quite distinguishable from the noise in the difference synthesis, and the occupancy factor for this atom becomes poorly determined. The amount of H in the H(03) site corresponds to  $0.75(10)$  H atoms per unit cell. The chemical analysis suggests a total hydrogen content of  $1.4$ ; thus at least half of the hydrogen in the cell is in the form of OH at the O(3) position, and is located such that a weak hydrogen bond occurs between  $\text{O}_3$  and  $\text{O}_5$ . The geometrical parameters of this hydrogen bond are  $\text{O}_3\text{—H } 0.94(5)$ ,  $\text{H} \dots \text{O}_5$   $2.46(5)$ ,  $\angle \text{O}_3\text{—H} \dots \text{O}_5$   $155^\circ(2)$ .<sup>1</sup>

<sup>1</sup> Hamilton and Ibers (1968) have given the geometrical criterion for a hydrogen bond as the existence of a heavy atom-hydrogen atom contact at least  $0.2 \text{ \AA}$  less than the sum of the van der Waals radii. (The last sentence in section 1.7 of this reference is unfortunately garbled.) Thus this would not be a hydrogen bond by the Hamilton-Ibers criterion. This is in agreement with the charge balance considerations of Donnay (1970).

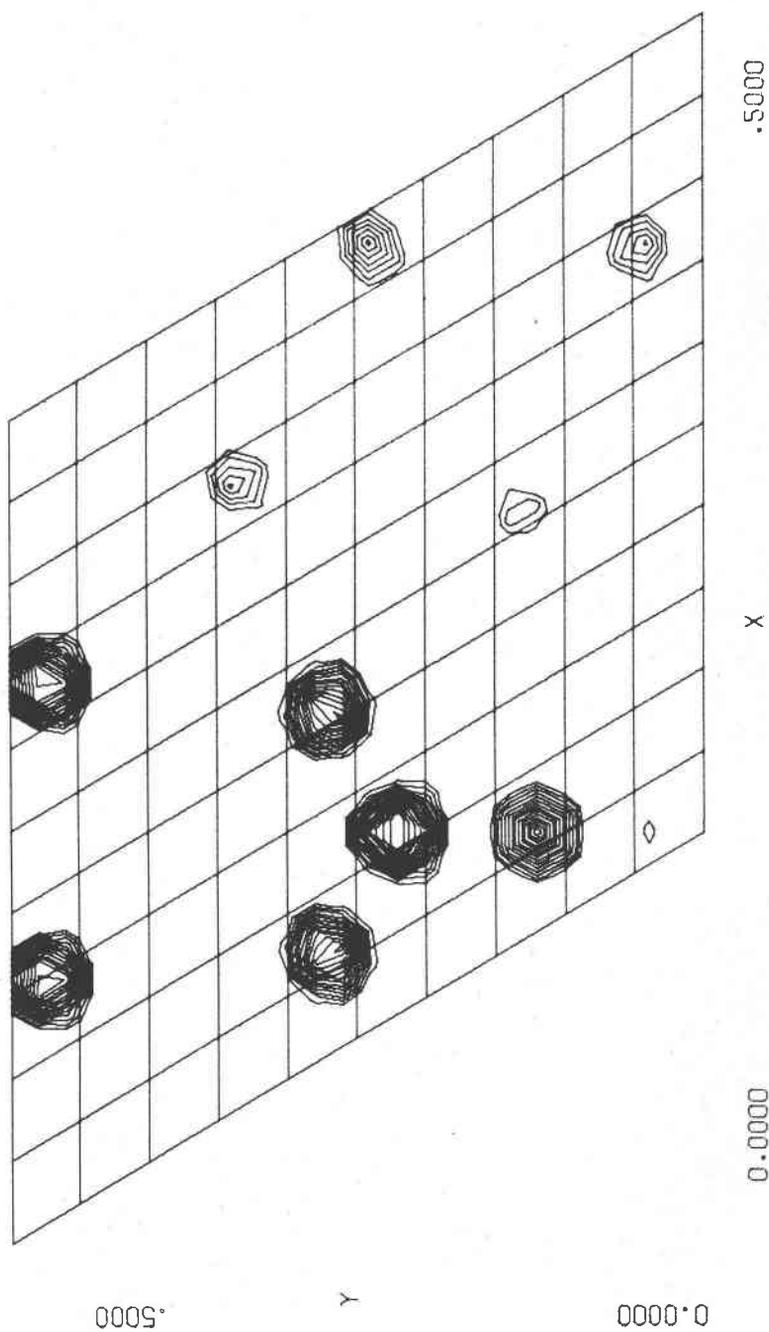


FIG. 1. Section ( $z = 0.4600$ ) of the three-dimensional neutron scattering density map for buergerite. This section shows the  $\text{BO}_3$  triangle clearly, as well as two  $\text{O}(6)$  atoms nearly in the plane. The smaller peaks are  $\text{O}(3)$ ,  $\text{O}(4)$ , and two  $\text{O}(7)$  atoms which do not lie in the plane of the section. The contour interval is  $0.225 \text{ fm} \cdot \text{\AA}^{-3}$ , and the minimum contour is at  $0.45 \text{ fm} \cdot \text{\AA}^{-3}$ .

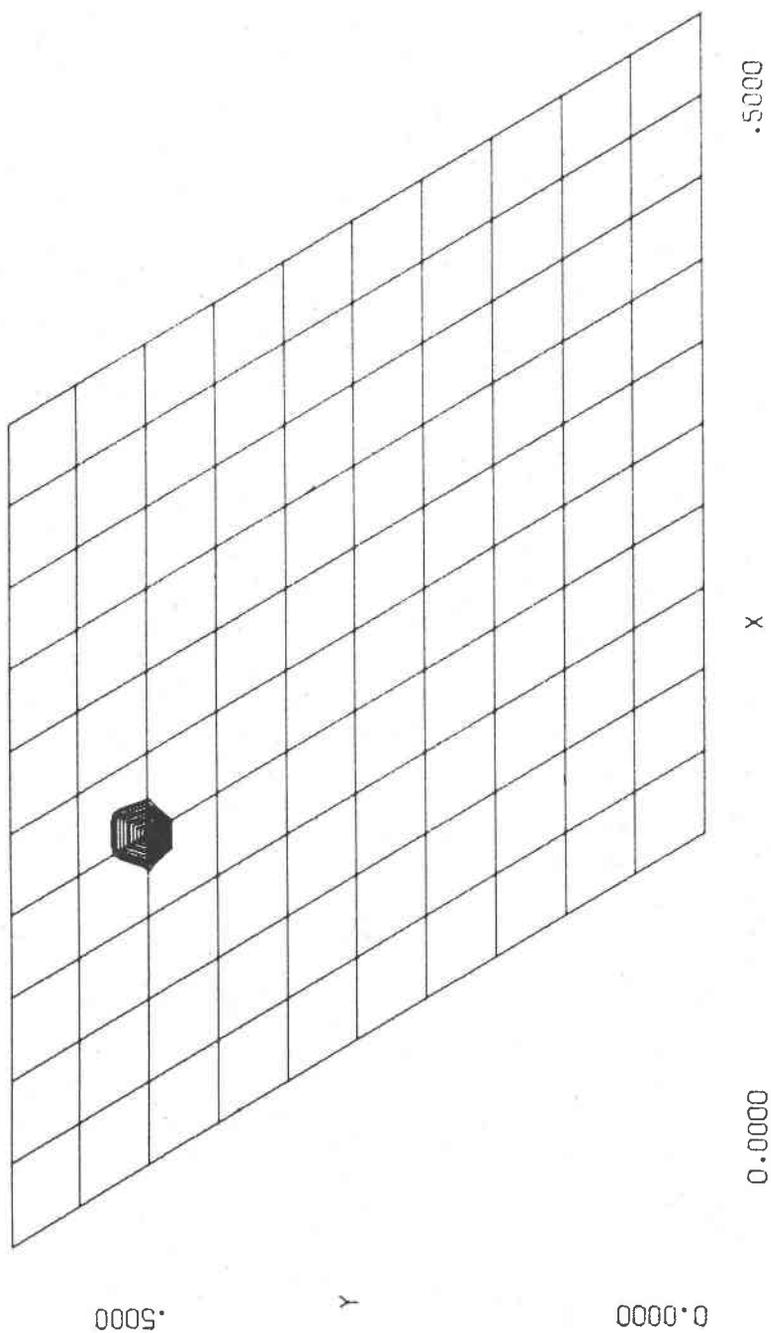


Fig. 2. Neutron scattering density map ( $z = 0.0600$ ) in the neighborhood of the hydrogen atom attached to O(3). The minimum contour is at  $-0.41 \text{ fm} \cdot \text{Å}^{-3}$ , the interval is  $-0.045 \text{ fm} \cdot \text{Å}^{-3}$ , and the hydrogen atom peak has a density of  $-0.91 \text{ fm} \cdot \text{Å}^{-3}$ .

A final cycle of least squares refinement, varying all parameters for the heavy atoms, but keeping an isotropic thermal parameter fixed at  $2.4 \text{ \AA}^2$  for H(O3), resulted in the structural parameters of Tables 2 and 3. The observed and calculated values of  $F^2$  are presented in Table 4.

TABLE 2. POSITIONAL PARAMETERS IN BUERGERITE  
Neutron parameters on first line, x-ray on second. Standard deviations in parentheses.

Atom	$x$	$y$	$z$
Na (3a)	0	0	0.21186 (139)
	0	0	0.21338 (41)
B (9b)	0.11001 (9)	-x	0.45172 (50)
	0.10992 (25)	-x	0.45182 (46)
Fe (9b)	-0.06614 (8)	-x	0.62217 (48)
	-0.06660 (4)	-x	0.62072 (10)
Si (18c)	0.19171 (17)	0.19087 (17)	0
	0.19156 (4)	0.19065 (4)	0
Al (18c)	0.29925 (19)	0.25919 (18)	0.60450 (54)
	0.29880 (4)	0.25887 (4)	0.60437 (10)
O(1) = F (3a)	0	0	0.76850 (74)
	0	0	0.76698 (51)
O(2) (9b)	0.06042 (9)	-x	0.48532 (54)
	0.06056 (15)	-x	0.48607 (29)
O(3) (9b)	-0.13205 (10)	-x	0.52026 (48)
	-0.13225 (17)	-x	0.52094 (30)
O(4) (9b)	0.09463 (9)	-x	0.07612 (50)
	0.09478 (15)	-x	0.07532 (28)
O(5) (9b)	-0.09117 (11)	-x	0.08362 (52)
	-0.09129 (16)	-x	0.08388 (28)
O(6) (18c)	0.19320 (13)	0.18712 (12)	0.77538 (41)
	0.19314 (11)	0.18680 (11)	0.77481 (18)
O(7) (18c)	0.28708 (12)	0.28592 (11)	0.07513 (44)
	0.28670 (10)	0.28580 (10)	0.07445 (18)
O(8) (18c)	0.20925 (13)	0.26971 (12)	0.43868 (43)
	0.20894 (10)	0.26941 (10)	0.43778 (20)
H (9b)	-0.1313 (19)	-x	0.3889 (72)

TABLE 3. THERMAL PARAMETERS<sup>a</sup> FOR BUERGERITE  
 Neutrons on first line, X-rays on second, standard deviations in parentheses

	$\beta_{11}$	$\beta_{22}$	$\beta_{33}$	$\beta_{12}$	$\beta_{13}$	$\beta_{23}$
Na	325 (29)	$\beta_{11}$	720 (163)	$\frac{1}{2}\beta_{11}$	0	0
	302 (14)	$\beta_{11}$	1172 (58)	$\frac{1}{2}\beta_{11}$	0	0
B	59 (12)	$\beta_{11}$	340 (50)	33 (11)	18 (10)	$-\beta_{13}$
	62 (16)	$\beta_{11}$	263 (40)	38 (10)	5 (17)	$-\beta_{13}$
Fe	103 (9)	$\beta_{11}$	585 (46)	15 (7)	87 (8)	$-\beta_{13}$
	72 (3)	$\beta_{11}$	544 (10)	3 (2)	87 (4)	$-\beta_{13}$
Si	63 (12)	62 (11)	294 (44)	29 (9)	- 5 (17)	-34 (17)
	40 (2)	38 (2)	152 (8)	18 (2)	3 (4)	- 8 (4)
Al	63 (15)	58 (14)	233 (50)	14 (10)	- 7 (20)	41 (19)
	64 (2)	86 (3)	286 (9)	34 (2)	- 7 (4)	35 (4)
O(1)=F	192 (15)	$\beta_{11}$	359 (85)	$\frac{1}{2}\beta_{11}$	0	0
	378 (24)	$\beta_{11}$	328 (66)	$\frac{1}{2}\beta_{11}$	0	0
O(2)	99 (9)	$\beta_{11}$	415 (45)	77 (10)	9 (9)	$-\beta_{13}$
	103 (14)	$\beta_{11}$	516 (36)	77 (8)	9 (14)	$-\beta_{13}$
O(3)	67 (9)	$\beta_{11}$	381 (49)	- 4 (10)	-11 (10)	$-\beta_{13}$
	82 (12)	$\beta_{11}$	367 (33)	14 (8)	8 (15)	$-\beta_{13}$
O(4)	86 (9)	$\beta_{11}$	356 (47)	33 (10)	-20 (10)	$-\beta_{13}$
	93 (13)	$\beta_{11}$	330 (33)	33 (8)	- 5 (15)	$-\beta_{13}$
O(5)	95 (8)	$\beta_{11}$	298 (41)	15 (10)	- 3 (9)	$-\beta_{13}$
	95 (13)	$\beta_{11}$	330 (32)	24 (8)	-12 (15)	$-\beta_{13}$
O(6)	71 (8)	79 (8)	202 (28)	22 (6)	1 (13)	- 6 (12)
	88 (6)	73 (6)	274 (20)	32 (5)	5 (10)	- 2 (9)
O(7)	61 (8)	47 (9)	315 (30)	12 (6)	1 (13)	-21 (13)
	81 (6)	69 (6)	237 (20)	28 (5)	-17 (9)	-23 (9)
O(8)	39 (9)	96 (8)	411 (31)	26 (7)	32 (13)	52 (13)
	62 (5)	87 (6)	381 (22)	40 (5)	- 1 (9)	23 (9)

<sup>a</sup> Debye-Waller factor is  $\exp \left\{ -\sum_{i,j} h_i h_j \beta_{ij} \right\}$

The final values of  $R$  and  $R_w$  (based on  $F^2$ ) are 0.034 and 0.056. The value of  $S$ , the standard deviation of an observation of unit weight, is 1.164. The values of  $\sigma$  assigned as discussed above are also tabulated in Table 4.



TABLE 5. SOME BOND-LENGTH COMPARISONS BETWEEN THE NEUTRON AND X-RAY STUDIES OF BUERGERITE

		Neutrons	X-rays
$Si_6O_{18}$	Si-O(4)	1.623 (2) Å	1.620 (2) Å
	Si-O(5)	1.628 (3)	1.626 (3)
	Si-O(6)	1.616 (3)	1.620 (2)
	Si-O(7)	1.605 (3)	1.602 (2)
$BO_3$	B-O(2)	1.384 (2)	1.379 (4)
	B-O(8)	1.367 (2)	1.364 (4)
	O(8)-O(2)	2.386 (2)	2.379 (3)
	O(8)-O(8)	2.361 (4)	2.356 (3)
$FeO_6$	Fe-O(1)	2.100 (3)	2.111 (4)
	Fe-O(2)	2.003 (2)	2.003 (2)
	Fe-O(3)	1.954 (2)	1.942 (3)
	Fe-O(6)	1.994 (2)	1.989 (2)

$\beta_{11}(F)$ ,  $P_n - P_x = 0.19(3)$ . The value of  $|P_x - P_n|/\sigma$  exceeded  $2\sigma$  for 4 of the 28 position parameters and for 10 of the 59 thermal parameters. The values of  $\Sigma(\Delta/\sigma)^2$  were 81.88 and 145.95 which may be tested as  $\chi^2$  with 28 and 59 degrees of freedom. Both values are significant—even if the large contributions of the two parameters cited above are omitted. Thus, there are small systematic differences between the two determinations; these are not however *chemically* significant.

Comparison of some bond lengths, as further examples of the good agreement between the two studies, are presented in Table 5a. We conclude that there are no essential differences in geometry between the two studies.

#### THE M-AL DISTRIBUTION

The values of the scattering lengths refined for the 18c Al and the 9b Fe point positions provide values for the occupancy factors of the ions in these sites. Some uncertainty in the chemical analysis reported by Barton (possibly due to occluded  $SiO_2$ ) should not affect the Fe/Al ratio, although it does affect the total amount of either. We therefore adopt as parameters to be determined: (Al), the total number of atoms of Al in the unit cell; (M), the total number of M atoms in the unit cell;  $x$ , the number of Al atoms in 18c; and  $y$ , the number of M atoms in 18c. We may write the following equations:

TABLE 6. DISTRIBUTION OF  $M$  AND Al BETWEEN THE  $9b$  AND  $18c$  POINT POSITIONS

	Neutrons	X-rays
Al $18c$	.936 (13)	.9125 (23)
$M$ $18c$	.064 (13)	.0875 (23)
Al $9b$	.251 (26)	.2961 (23)
$M$ $9b$	.749 (26)	.251 (26)

$$0.35x + 0.898y = 18 \cdot 0.835 = 6.930 \pm 0.13$$

$$0.35[(Al) - x] + 0.898(0.414(Al) - y) = 9 \cdot 0.763 = 6.867 \pm 0.10.$$

(The  $M/Al$  ratio from the chemical analysis is 0.414.) Solution of these equations gives the following results:

$$(Al) = 19.11(22)$$

$$(M) \equiv 0.414(Al) = 7.91(8)$$

for a total of 27.0 atoms and no vacancies in either point position.

$$x = 16.85(23)$$

$$y = 1.15(24)$$

The population in  $9b$  is accordingly



These values agree, within two standard deviations, with the values determined by Barton. The percentage occupancies are given in Table 6. The agreement is again reasonable. It should be noted that Barton's quoted standard deviations do not take into account the considerable uncertainties in these values due to the uncertainties of the chemical analysis; his  $9b$   $M$  occupancy changed from 0.78 to 0.70 on revision of the chemical analysis.

#### MAGNETIC STRUCTURE

Magnetic susceptibility measurements (Tsang, Thorpe, Senftle, and Donnay, 1970) indicate that buergerite has an exchange constant  $J/k = 7.5^\circ K$ . Donnay *et al.* (1967) have suggested that antiferromagnetic ordering in tourmalines could consist of a trigonal arrangement of spins on iron atoms lying in the mirror plane. X-ray data indicate no structural transition down to  $8^\circ K$ .

We have measured neutron-diffraction powder patterns of buergerite at  $295^\circ K$  and  $4.2^\circ K$ . The patterns are essentially identical (Figure 3). No

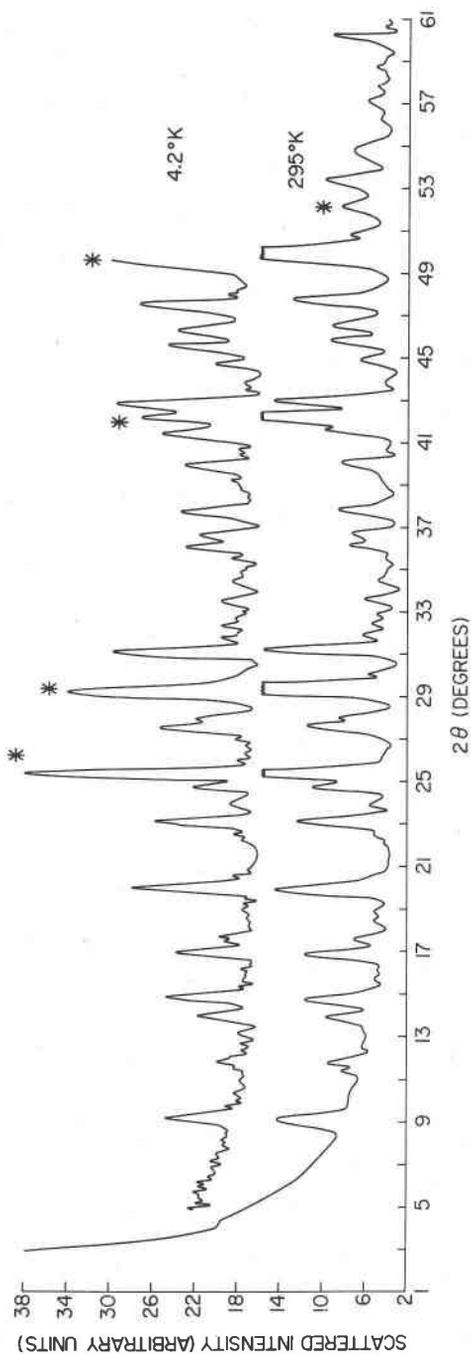


FIG. 3. Neutron powder patterns of buergerite at 4.2°K and 295°K. The low temperature pattern has been given an arbitrary vertical displacement; this pattern was run more quickly and hence has a greater noise level in the background. The peaks marked with asterisks are Al powder lines from the sample holder and cryostat. The neutron wavelength was 1.026 Å.

new peaks appear, and there are no significant changes in the intensity of any peaks. We conclude that there is no significant long range magnetic order in buergerite down to 4.2°K. Such order should give rise to appreciable changes in scattered neutron intensity in the Bragg peaks. The observation that there is no change in this intensity does not preclude *local* spin ordering of the type proposed by Donnay *et al.*

## ACKNOWLEDGEMENTS

We thank Dr. Gabrielle Donnay for stimulating our interest in this problem and Dr. John S. White of the Smithsonian Institution for supplying samples of buergerite for these studies.

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*Manuscript received, July 6, 1970; accepted for publication, July 30, 1970.*